Goldstein 7.19

$$(m_\pi c, 0) \longrightarrow (\gamma m_\mu c, \gamma m_\mu v)$$
$$+ (-\gamma m_\mu v, -\gamma m_\mu v)$$

The frame did not change, impose conservation of momentum:

$$m_\pi c = \gamma m_\mu c - \gamma m_\mu v = \gamma m_\mu [c - v].$$

$$\Rightarrow \quad \frac{m_\pi}{m_\mu} = \gamma [1-\beta]$$

$$\Rightarrow \quad \gamma \frac{m_\pi}{m_\mu} = \gamma^2 [1-\beta] = \frac{1}{(1-\beta)(1+\beta)} (1-\beta) = \frac{1}{1+\beta},$$

$$\frac{m_\mu}{m_\pi} = \gamma [1+\beta].$$

$$\Rightarrow \quad \frac{m_\pi}{m_\mu} + \frac{m_\mu}{m_\pi} = 2\gamma, \quad \gamma = \frac{1}{2}\left[\frac{m_\pi}{m_\mu} + \frac{m_\mu}{m_\pi}\right].$$

$$T = E - mc^2 = (\gamma - 1) m_\mu c^2.$$

$$= \left\{\frac{1}{2}\left[\frac{m_\pi}{m_\mu} + \frac{m_\mu}{m_\pi}\right] - 1\right\} m_\mu c^2$$

$$= \left[\frac{m_\pi^2 + m_\mu^2}{2 m_\pi m_\mu} - 1\right] m_\mu c^2$$

$$= \boxed{\frac{(m_\pi - m_\mu)^2}{2 m_\pi} c^2}$$

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1.14.2024